An Efficient Preprocessing Approach for Uncertainty Consideration in Microgrids

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***Abstract*— Uncertainty considerations in microgrid operation and planning are of significant importance as uncertain factors can potentially alter the operator's decisions. New mathematical approaches, such as robust optimization, are commonly adopted to capture uncertainties and ensure practicality. However, this added practicality is at the expense of increased problem size and computational complexity. This paper presents a detailed discussion and analysis of prevailing uncertainties in microgrid operation and planning, and accordingly proposes a new preprocessing approach to integrate uncertainties while reducing computational requirements. Numerical simulations exhibit the merits of the proposed approach over the commonly used robust optimization method from the execution time and practicality perspectives.**

***Index Terms*—Distributed energy resource, microgrid, preprocess, robust optimization, uncertainty.**

NOMENCLATURE

c	Generation price for dispatchable units
$C^{\max}$	Rated capacity of energy storage systems
ch	Superscript for energy storage charging mode
dch	Superscript for energy storage discharging mode
D	Set of dual variables
D	Load demand
g	Superscript for uncertain renewable generation
G	Set of dispatchable units
i	Index for DERs
l	Superscript for the uncertain load
LS	Load curtailment
P	Set of primal variables
P	DER output power
P_M	Main grid power
$P^{\max}$	Rated power of DERs
$P_M^{\max}$	Flow limit between microgrid and the main grid
S	Set of energy storage systems
t	Index for hour
u	Auxiliary binary variables for uncertain parameters
u_M	Binary islanding variable
U	Set of uncertain parameters
v	Value of lost load (VOLL)
W	Set of nondispatchable units
x	Uncertain parameter
$\wedge$	Index for calculated/given variables
$\sim$	Index for forecasted parameters
ρ	Market price
η	Energy storage efficiency
Γ	Limit on uncertainty option
$\lambda, \pi, \vartheta, \mu, \psi, \xi, \theta$	Dual variables

I. INTRODUCTION

MICROGRIDS, as promoters of pervasive distributed generation, improved grid reliability, and the greener energy economy, have been significantly deployed over the past few years and are anticipated to grow even more in the near future [1]. During the past decade, a significant amount of research has been devoted to study microgrids and to facilitate development and implementation efforts. The number of published articles focused on microgrids has been tripled over the past five years and microgrid deployments have been federally supported in the United States, particularly by establishing the U.S. DOE Microgrid Initiative [2]. Over the 2011 to 2014 period, there have been numerous microgrid projects in the United States funded by DOE, DOD, industry and governmental labs matching funds, as well as electric utilities, totaling more than $213 million. Similar trend can also be seen globally where the 2013 worldwide installed microgrids generation capacity exceeded 4.1 GW [3]. These figures clearly represent the growing interest in this new technology and picture future power grids as systems of interconnected microgrids. Microgrids are more than just backup generation, as they could efficiently manage a set of local generation and load resources and introduce unique operational opportunities for local customers, such as improved reliability, higher power quality, economic operation, and offering energy efficiency [4]-[12].

One important issue in managing microgrids is the role of uncertainties. Uncertainty represents factors, which having a major influence on scheduling decisions, are out of control of the microgrid controller and/or cannot be forecasted with certainty. Uncertainty considerations in power system operation and planning have been significantly increased in the past few years. Two common approaches for considering uncertainty are stochastic programming and robust optimization. Stochastic models are commonly based on sampling methods with pre-assumed probability distribution functions, which convert the original objective to the weighted average of objectives for individual scenarios. However, a concrete characterization of the uncertainty requires a large number of scenarios, especially when uncertainties are not discrete. Thus, the derived large-scale stochastic problem is more time-intensive and considerably harder to solve than the original problem. In addition, probability distributions cannot be accurately estimated which would obstruct the practical implementation of this technique. On the other hand, in robust optimization, each uncertain parameter is associated with an uncertainty interval, i.e., an upper bound and a lower bound, where the optimization problem ensures the feasibility of the solution in the worst-case scenarios [13]. Thus, in contrast with stochastic programming, there is no need to accurately determine distribution probability functions related to uncertain data. Furthermore, the robust optimization problem does not suffer from the curse of dimensionality since only one robust problem is solved rather than a set of problems corresponding to individual scenarios. However, the robust optimization solution is obtained at the expense of sacrificing a certain level of the solution optimality and increased computational complexity.

In this paper, a preprocess approach is proposed to identify uncertainties that result in the robust (i.e., worst-case) solution. In other words, the solution of the robust optimization will be achieved without the need to solve the robust problem. Using this preprocess approach, the primal microgrid operation problem, which is linear and convex, can be solved instead of the dual problem that is required in the robust optimization and contains a large number of binary variables, hence addressing the computational complexity problem. This paper performs studies on the microgrid optimal scheduling problem which also acts as a core component in longer term maintenance and planning problems.

The rest of the paper is organized as follows. Section II discusses microgrid's prevailing uncertainties and further develops the microgrid optimal scheduling model. Section III discusses the possibility of determining uncertainties as a preprocess. Section IV provides numerical simulations for a test microgrid to validate findings and evaluate the proposed approach. The discussions on the proposed approach and conclusions are provided in Sections V and VI, respectively.

II. Microgrid Optimal Scheduling

A. Discussion on Uncertainties

Uncertainties involved in the microgrid optimal scheduling can be attributed into two groups of forecasting-related and islanding-related. Forecast errors represent uncertainties in accurately forecasting future values of microgrid load, variable renewable generation, and time-dependent market prices. The islanding-related uncertainty represents the uncertain time and duration of main grid outages in which the microgrid needs to operate in the islanded mode. An extensive discussion on uncertainties in microgrids can be found in [13]. This paper only focuses on the forecasting-related uncertainty.

B. Microgrid Optimal Scheduling Under Uncertainties

The day-ahead microgrid optimal scheduling problem is formulated as follows.

$$\max_{\mathrm{U}}\min_{\mathrm{P}} \sum_t\sum_{i\in \mathrm{G}} c_i P_{it} + \sum_t \rho_t P_{M,t} + \sum_t v_t LS_t \tag{1}$$

$$\sum_{i\in\{\mathrm{G,W}\}} P_{it} + \sum_{i\in \mathrm{S}} (P_{it}^{dch} - P_{it}^{ch}) + P_{M,t} + LS_t = D_t \quad \forall t \tag{2}$$

$$-P_M^{\max} u_{M,t} \le P_{M,t} \le P_M^{\max} u_{M,t} \qquad \forall t \tag{3}$$

$$0 \le P_{it} \le P_i^{\max} \qquad \forall i \in \mathrm{G}, \forall t \tag{4}$$

$$P_{it} = \hat{P}_{it} \qquad \forall i \in \mathrm{W}, \forall t \tag{5}$$

$$0 \le P_{it}^{dch} \le P_i^{\max} \qquad \forall i \in \mathrm{S}, \forall t \tag{6}$$

$$0 \le P_{it}^{ch} \le P_i^{\max} \qquad \forall i \in \mathrm{S}, \forall t \tag{7}$$

$$0 \le \sum_{\tau\le t} (P_{i\tau}^{ch} - P_{i\tau}^{dch}/\eta_i) \le C_i^{\max} \qquad \forall i \in \mathrm{S}, \forall t \tag{8}$$

$$0 \le LS_t \le D_t \qquad \forall t \tag{9}$$

The objective of the optimal scheduling problem is to minimize the microgrid operation cost (1), including the generation cost of dispatchable units, the cost of energy purchase from the main grid, and the cost of unserved energy. The objective is maximized over uncertainty sets to achieve the worst-case microgrid optimal operation solution. The cost of unserved energy is defined as the load curtailment quantity multiplied by the value of lost load (VOLL), where higher VOLLs represent more critical loads [14]. The load balance equation (2) ensures that the sum of power generated by all DERs, including dispatchable and nondispatchable units as well as energy storage, and power from the main grid is equal to the hourly load. Additional operational constraints include the limit the amount of exchanged power with the main grid (3), dispatchable units' generation capacity limits (4), nondispatchable units generation (5), the energy storage charging and discharging limits (6)-(7), the energy storage available energy (8), and the limit on load curtailments (9). A binary islanding parameter is added to (3) to model grid-connected ($u_{M,t}$=1) and islanded ($u_{M,t}$=0) operation modes. Since line flows are relatively small, the distribution network congestion is neglected. The proposed microgrid optimal scheduling model is developed in a linear format where the binary variables associated with the commitment state of dispatchable units and charging/discharging states of energy storage are ignored.

To solve the proposed microgrid optimal scheduling problem, in which its objective has a max-min format, the dual problem of the inner minimization problem is combined with the outer maximization problem. The resultant problem with dual variables and uncertain parameters is as follows:

$$\begin{aligned}\max_{\mathrm{U,D}} & \sum_t \lambda_t \hat{D}_t + \sum_t (\mu_t^- + \mu_t^+) P_M^{\max} u_{M,t} \\ & + \sum_t\sum_{i\in \mathrm{W}} \vartheta_{it}\hat{P}_{it} + \sum_t\sum_{i\in \mathrm{G}} \pi_{it} P_i^{\max} \\ & + \sum_t\sum_{i\in \mathrm{S}} \psi_{it}^{dch} P_i^{\max} + \sum_t\sum_{i\in \mathrm{S}} \psi_{it}^{ch} P_i^{\max} \\ & + \sum_t\sum_{i\in \mathrm{S}} \xi_{it}^{+} C_i^{\max} \\ & + \sum_t \theta_t \hat{D}_t\end{aligned} \tag{10}$$

$$\lambda_t + \pi_{it} \le c_i \qquad \forall i \in \mathrm{G}, \forall t \tag{11}$$

$$\lambda_t + \vartheta_{it} = 0 \qquad \forall i \in \mathrm{W}, \forall t \tag{12}$$

$$\lambda_t + \psi_{it}^{dch} - \sum_{\tau\ge t} (\xi_{i\tau}^{+} - \xi_{i\tau}^{-})/\eta_i \le 0 \qquad \forall i \in \mathrm{S}, \forall t \tag{13}$$

$$-\lambda_t + \psi_{it}^{ch} + \sum_{\tau\ge t} (\xi_{i\tau}^{+} - \xi_{i\tau}^{-}) \le 0 \qquad \forall i \in \mathrm{S}, \forall t \tag{14}$$

$$\lambda_t + (\mu_t^+ - \mu_t^-) = \rho_t \qquad \forall t \tag{15}$$

$$\lambda_t + \theta_t \le v_t \qquad \forall t \tag{16}$$

where λ, μ, π, υ, ψ^{dch}, ψ^{ch}, ξ, and θ are dual variables of constraints (2)-(9), respectively. Considering polyhedral uncertainty sets, and assuming that the worst-case solution occurs at extreme points, uncertain parameters can be represented as a function of the nominal forecasted value and the uncertainty interval with the aid of auxiliary binary variables. For example, the uncertain parameter x can be written as $x = \tilde{x}_t - \underline{x}_t \underline{u}_t + \overline{x}_t \overline{u}_t$ where inserted bars represent its upper/lower bounds. To prevent simultaneous occurrence of extreme points, one binary variable can be set at one at any given hour, i.e., $\underline{u}_t + \overline{u}_t \leq 1$. Compared to the primal problem which was a linear problem, a large amount of binary variables will be added to the robust problem. The addition of binary variables would create a nonlinear and computationally more challenging optimization problem. Bilinear terms, as would appear in (10) when binary variables are used, should be further converted into linear terms which would accordingly add additional variables to the problem [15].

C. Uncertainty Control

The level of solution conservatism can be efficiently controlled by limiting the total number of uncertain parameters that can reach their extreme values, or in other words, the total number of binary auxiliary variables that can reach a value of 1. The limit on uncertainty options is given in (17). The larger the limit on uncertainty option, a more robust solution is obtained against uncertainties, resulting in a larger operation cost. On the other hand, the smaller the limit on uncertainty option, a more aggressive solution is obtained, resulting in a less robust solution. A moderate solution considers some level of uncertainty in between.

$$\sum_t (\underline{u}_t + \overline{u}_t) \leq \Gamma \tag{17}$$

The limit on uncertainty option is a necessary tool to control the solution conservatism and prevent large deviations from the optimal solution. This limit, however, adds additional computational complexity to the problem as only a selected set of binary variables can reach a value of 1. To address the computational complexity, a preprocessing approach, as discussed in the next section, is proposed.

III. Proposed Preprocessing Approach

The objective of the proposed preprocessing approach is to determine uncertainties without the need to solve the computationally challenging robust optimization problem developed in Section II. To perform preprocessing, first a set of efficient signals for each type of uncertainty should be developed as discussed in the following:

Load signal: Considering the proposed uncertainty definition, the load uncertainty will be defined as $D = \widetilde{D}_t - \underline{D}_t \underline{u}_t^l + \overline{D}_t \overline{u}_t^l$ and the corresponding term in the objective function (10) would be $\sum_t (\lambda_t + \theta_t)(\widetilde{D}_t - \underline{D}_t \underline{u}_t^l + \overline{D}_t \overline{u}_t^l)$. It can be shown that λ_t, i.e., the dual variable associated with the load balance constraint (2), is always positive in the proposed robust problem, and also θ_t is zero in grid-connected modes as there will be no load curtailments. Therefore, demand will maximize the objective (10) when it is larger than the forecasted value, or equivalently, when it is at its upper bound, i.e., $\overline{u}_t^l = 1$ and $\underline{u}_t^l = 0$. The uncertain load will be accordingly represented by $\widetilde{D}_t + \overline{D}_t$. Considering that the upper and lower bounds of the uncertainty interval are linear functions of the nominal value, e.g., $\overline{D}_t = 0.1 \times \widetilde{D}_t$ for a 10% forecast error, the load signal of $\lambda_t(\widetilde{D}_t + \overline{D}_t)$ will be considered for characterizing the load uncertainty. By calculating this signal and sorting values from the highest to the lowest, the order of hours of the day in which the worst-case load has happened can be efficiently determined.

Renewable generation signal: The renewable uncertainty will be defined as $P_{it} = \tilde{P}_{it} - \underline{P}_{it} \underline{u}_{it}^g + \overline{P}_{it} \overline{u}_{it}^g$. It can be shown that ϑ_{it}, i.e., the dual variable associated with the generation of renewable sources (5), is always negative in the proposed robust problem. Therefore, variable renewable sources will maximize the objective (10) when they generate less than the forecasted value, or equivalently, when reaching the lower bound, i.e., $\underline{u}_{it}^g = 1$ and $\overline{u}_{it}^g = 0$. The power generated by variable renewable sources will be accordingly represented by $\tilde{P}_{it} - \underline{P}_{it}$. A lower value for $\vartheta_{it}(\tilde{P}_{it} - \underline{P}_{it})$ will result in a larger impact on the objective value, hence this term will be considered as the signal to determine the worst-case scenario of uncertainties in renewable generation. By calculating this signal and sorting values from the lowest to the highest, the order of hours of the day in which the worst-case has happened would be determined.

Market price signal: The worst-case scenario of uncertainties in market prices depends on the microgrid power exchange with the main grid, i.e., selling or buying. If the microgrid is selling power in a specific hour, i.e., negative exchange power with the main grid, the worst-case in that hour would occur at the lower bound in which the market price is less than the forecasted value. Similarly, if the microgrid is buying power in a specific hour, i.e., a positive exchange power with the main grid, the worst-case in that hour would occur at the upper bound in which the market price is more than the forecasted value. By changing market prices, generation prices of dispatchable units should be noted. If the market price in a specific hour is less than the generation price of a dispatchable unit, the microgrid would prefer to buy power from the main grid instead of dispatching that unit, therefore $P_{M,t}$ would be positive. The worst-case in this situation would occur when the market price is increased. If the market price in that hour increases to the extent that it becomes higher than the generation price of the dispatchable unit, the microgrid would prefer to dispatch that unit and sell power to the main grid. On the other hand, if the market price in a specific hour is higher than the generation price of a dispatchable unit, the microgrid would prefer to dispatch that unit and sell power to the main grid, therefore $P_{M,t}$ would be negative. The worst-case in this situation would occur when the market price is further decreased. As a result, the signal for measuring the uncertainty in market price would comprise two parts; one is the effect of the exchange power and change in the market price, and the other is the effect of changes in the market price on turning dispatchable units on or off. In summary and based on the discussions, $\Delta\rho_t . P_{M,t} + \Delta P_{it} . (c_i - \rho_t - \Delta\rho_t)$ could be considered as a signal to determine the worst-case scenario of uncertainties in market prices. Again, by calculating the proposed signal and sorting values from the highest to the lowest, the order of hours of the day in which the worst-case has happened would be determined.

IV. Numerical Simulations

A microgrid is installed for a group of electricity customers with a peak load demand of 17 MW. The set of DERs used in this study includes four dispatchable units, one wind unit, one solar unit, and one energy storage [11]. The cost coefficients of dispatchable units 1-4 are considered to be \$27.7/MWh, \$39.1/MWh, \$61.3/MWh, and \$65.6/MWh, respectively. The load, renewable energy, and market price are forecasted based on historical data obtained from the Illinois Institute of Technology Campus Microgrid [16]. Data of storage, wind, and solar are gathered from [17] and [18]. The efficiency of the energy storage and the VOLL are considered to be 100% and \$10,000/MWh, respectively. The upper and lower bounds for all sources of uncertainty are considered to be 10% of the forecasted data. The microgrid optimal scheduling problem is implemented on a high performance computing server consisting of four 10-core Intel Xeon E7-4870 2.4 GHz processors with 128 GB memory. The problem was formulated by mixed-integer programming (for the robust optimization problem) and linear programming (for the primal problem in the proposed approach) and solved by CPLEX 12.6 [19]. Two cases are studied to validate the accuracy of the proposed approach as well as its impact on reducing the computational complexity.

Case 1 (Validation): The proposed preprocess approach in uncertainty consideration is applied to the test microgrid to ensure its viability in identifying uncertainties for loads, renewable generation, and market prices. By increasing the budget of uncertainty option in the load from 0 to 24 and solving the dual problem, the order of hours that cause the worst-cases would be 17, 18, 19, 20, 16, 21, 14, 15, 22, 13, 12, 23, 24, 11, 10, 8, 9, 6, 7, 5, 4, 1, 3, and 2. The calculations of the proposed signal for load uncertainties, i.e., $\lambda_t(\widetilde{D}_t + \overline{D}_t)$, are shown in Fig. 1. By sorting the calculated values in all hours, it can be seen that the results would be the same as those obtained by solving the dual problem, meaning that the term $\lambda_t(\widetilde{D}_t + \overline{D}_t)$ would be a proper signal to assess the load uncertainty. Similarly, for the renewable generation, by increasing the budget of uncertainty option in renewable generation units from 0 to 24 and solving the dual problem, the order of hours that cause the worst-case realization with respect to the wind generation would be 21, 22, 13, 12, 14, 11, 6, 8, 9, 5, 7, 10, 17, and 18. Similarly, the order of hours that cause the worst-case realization with respect to the solar generation would be 17, 16, 18, 20, 15, 14, 19, 13, and 12. The wind and solar generation in other hours is zero. The calculations of the proposed signal for wind and solar uncertainties, i.e., $\vartheta_{it}(\tilde{P}_{it} - \underline{P}_{it})$, are shown in Fig. 2. By sorting the calculated values in all hours, it can be seen that the results would be the same as those obtained by solving the dual problem, meaning that the term $\vartheta_{it}(\tilde{P}_{it} - \underline{P}_{it})$ would be a proper signal to assess the renewable generation uncertainty.

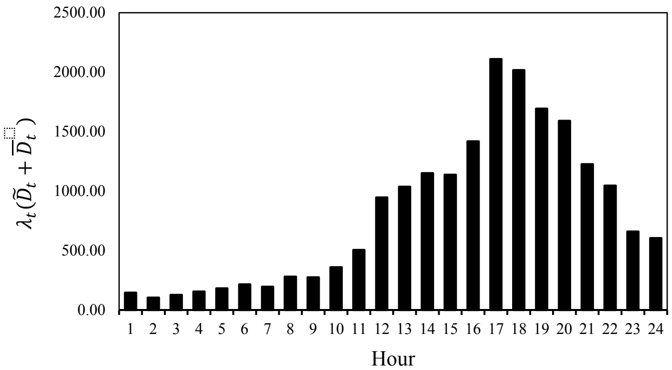

Fig. 1. Impact of the proposed signal for load uncertainties

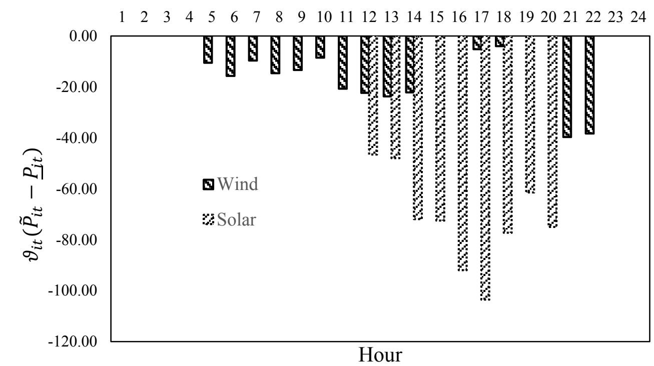

Fig. 2. Impact of the proposed signal for renewable uncertainties

For the market price uncertainty, first it is assumed that there is not any storage unit. By increasing the budget of uncertainty option in market prices from 0 to 24 and solving the dual problem, the order of hours that cause the worst-case realization would be 12, 22, 21, 8, 9, 10, 6, 7, 16, 4, 5, 11, 1, 20, 17, 3, 19, 2, 18, 24, and 13. The calculated signal for market price uncertainties, i.e., $\Delta\rho_t.P_{M,t} + \Delta P_{it}.(c_i - \rho_t - \Delta\rho_t)$, is shown in Fig. 3. It should be noted that 10% change in the market price would cause dispatchable unit 1 in hour 10, unit 2 in hour 11, unit 4 in hour 15, and unit 3 in hour 23 to be turned on. It also causes dispatchable unit 4 in hours 12 and 22 and also units 3 and 4 in hours 13 and 14 to be turned off. Therefore, as discussed in Section III, the second term of the proposed signal, i.e., $\Delta P_{it}.(c_i - \rho_t - \Delta\rho_t)$, should be considered for calculations at the aforementioned hours. By sorting the calculated values in all hours, it can be seen that the results would be the same as those obtained by solving the dual problem, meaning that $\Delta\rho_t.P_{M,t} + \Delta P_{it}.(c_i - \rho_t - \Delta\rho_t)$ is a proper signal to assess the market price uncertainty. By considering energy storage in the assessment of market price uncertainties, the results calculated by the signal are slightly different from those obtained by solving the dual problem. However, the differences are marginal and can be ignored with acceptable accuracy.

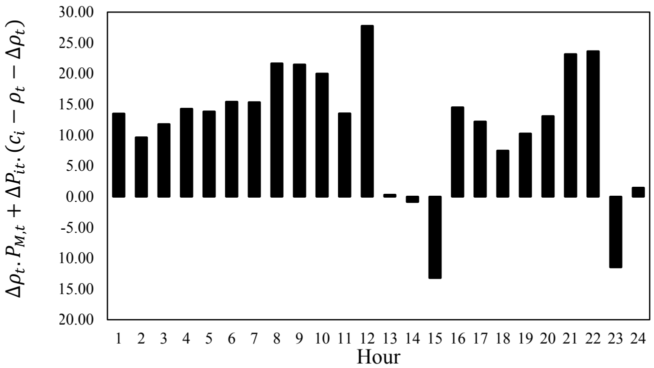

Fig. 3. Impact of the proposed signal for market price uncertainties

TABLE I COMPARISON BETWEEN THE ROBUST OPTIMIZATION PROBLEM AND THE PROPOSED PREPROCESSING APPROACH

	Robust optimization with dual variables	Proposed uncertainty preprocessing
No. of continuous variables	201,480	87,600
No. of binary variables	70,080	0
No. of constraints	411,725	122,640
Computation time	2.5-4 hours	~20 seconds

Case 2 (Evaluation): The optimal scheduling problem is formulated using MIP and extended to obtain a one-year planning problem based on the data in [13]. There are eight sets of binary variables in the robust problem associated with upper and lower bounds of uncertainty intervals: two sets for the load, two sets for each of the two renewable generation units, and two sets for market prices. Each variable should be defined at every single hour during the scheduling period, therefore there would be 70,080 (= 8×8760) binary variables which should be determined in order to find the worst-case realization. Such a large number of binary variables would considerably increase the computational complexity. The number of binary variables will also be further larger when: 1) a longer planning time horizon, e.g., 20 years, is considered, and 2) a shorter operation time period, e.g., 10-min operation to capture renewable generation variability instead of the hourly scheduling, is considered. In either case, the obtained robust problem will be significantly larger and noticeably more difficult to solve considering the large number of added binary variables.

The comparison between the two methods for a one-year planning problem is shown in Table I. The proposed method in this paper, which introduces signals to determine uncertainties, does not employ binary variables and formulates the problem using linear programming. The results show that reducing the number of variables and constraints would significantly decrease the computation time from 2.5-4 hours to less than a minute.

V. CONCLUSIONS

This study presented a detailed discussion and analysis of uncertainties in the microgrid optimal scheduling problem. The least-cost operation objective was maximized over uncertainty sets, using robust optimization, to achieve the worst-case optimal solution in the microgrid day-ahead operation and accordingly capture forecast uncertainties. To address the computational complexity associated with the robust optimization model, a preprocess approach was proposed which was capable of identifying uncertainties without the need to formulate and solve the robust problem. Instead, the preprocessing approach relied on solving the original linear problem and accordingly creating a set of uncertainty signals to identify the worst-case realizations of uncertain parameters. Based on the proposed preprocess approach, it was shown that the worst-case realization for load would occurred at its upper bound, and for renewable generation at its lower bound. The worst-case realization for the market prices were contingent on whether the microgrid was selling power to or buying power from the utility grid. Numerical examples demonstrated that the proposed signals can accurately determine worst-case realization in load, renewable generation, and market prices, and the proposed approach was capable of significantly reducing the complexity and the computation time of microgrid operation and planning problems under uncertainty.

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